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## False vacuum decay: effective one-loop action for pair creation of domain walls

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### Abstract

An effective one-loop action built from the soliton field itself for the two-dimensional (2D) problem of soliton pair creation is proposed. The action consists of the usual mass term and a kinetic term in which the simple derivative of the soliton field is replaced by a covariant derivative. In this effective action the soliton charge is treated no longer as a topological charge but as a Noether charge. Using this effective one-loop action, the soliton-antisoliton pair production rate  $\Gamma/L = A \exp[-S_0]$  is calculated and one recovers Stone's exponential factor  $S_0$  and the prefactor  $A$  of Kiselev, Selivanov and Voloshin. The results are also valid straightforwardly to the problem of pair creation rate of domain walls in dimensions  $D \geq 3$ .

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## I. Introduction

Stone [1] has studied the problem of a scalar field theory in (1+1)D with a metastable vacuum, i.e., with a scalar potential that has a false vacuum,  $\phi_+$ , and a true vacuum,  $\phi_-$ , separated by an energy density difference,  $\epsilon$ . Stone has noticed that the decay process can be interpreted as the false vacuum decaying into the true vacuum plus a creation of a soliton-antisoliton pair:  $\phi_+ \rightarrow \phi_- + s + \bar{s}$ . The energy necessary for the materialization of the pair comes from the energy density difference between the two vacua. The soliton-antisoliton pair production rate per unit time and length,  $\Gamma/L$ , can then be identified with the decay rate of the false vacuum and is given by ( $\hbar = c = 1$ ) [1]:

$$\Gamma/L = A e^{-S_0} = A e^{-\frac{\pi m^2}{\epsilon}}, \quad (1)$$

where  $m$  is the soliton mass and prefactor  $A$  is a functional determinant whose value was first calculated by Kiselev and Selivanov [2, 3] and later by Voloshin [4]. Extensions to this decay problem, such as induced false vacuum decay, have been studied by several authors (for a review and references see, e.g., [5, 6]).

The method used in [1, 2, 3, 4] is based on the instanton method introduced by Langer in his work about decay of metastable thermodynamical states [7]. This powerful method has been applied to several different studies, namely: Coleman and Callan [8, 9] have computed the bubble production rate that accompanies the cosmological phase transitions in a (3+1)D scalar theory (this was indeed previously calculated by other methods by Voloshin, Kobzarev and Okun [10]); Affleck and Manton [11] have studied monopole pair production in a weak external magnetic field and Affleck, Alvarez and Manton [12], have studied  $e^+e^-$  boson pair production in a weak external electric field. Recent developments studying pair production of boson and spinorial particles in external Maxwell's fields have been performed by several authors using different methods [13]-[15] and similar results in the Euler-Heisenberg theory, a modified Maxwell theory, have been also obtained [16]. The decay of false vacuum in a condensed matter system providing soliton tunneling has been studied in [17, 18].

In this paper we propose an effective one-loop action built from the soliton field itself to study the problem of Stone [1], Kiselev and Selivanov [2, 3] and Voloshin [4]. The action consists of the usual mass term and a kinetic term in

which the simple derivative of the soliton field is replaced by a kind of covariant derivative. In this effective action the soliton charge is treated no longer as a topological charge but as a Noether charge. This procedure of working with an effective action for the soliton field itself has been introduced by Coleman [19] where the equivalence between the Sine-Gordon model and the Thirring model was shown, and by Montonen and Olive [20] who have proposed an equivalent dual field theory for the Prasad-Sommerfield monopole soliton. More connected to our problem, Manton [21] has proposed an effective action built from the soliton field itself which reproduces the soliton physical properties of (1+1)D nonlinear scalar field theories having symmetric potentials with degenerate minima. In this paper we deal instead with a potential with non-degenerate minima in a (1+1)D scalar field theory. Thus, our effective action is new since Manton was not dealing with the soliton pair production process.

Using the effective one-loop action and the method presented in [12], we calculate the soliton-antisoliton pair production rate, (1). One recovers Stone's exponential factor  $S_0$  [1] and the prefactor  $A$  of Kiselev and Selivanov [2, 3] and Voloshin [4].

## II. Effective one-loop action

In order to present some useful soliton properties let us consider a scalar field theory in a (1+1)D spacetime, whose dynamics is governed by the action (see, e.g., [22]),

$$S[\phi(x, t)] = \int d^2x \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) \right], \quad (2)$$

where  $U$  is a generic potential. A particular important case is when  $U$  is a symmetric potential,  $U = U_s(\phi)$ , with two or more degenerate minima. In the  $\phi^4$  theory the potential is  $U_s(\phi) = \frac{1}{4} \lambda (\phi^2 - \mu^2/\lambda)^2$ , with  $\mu \geq 0$  and  $\lambda \geq 0$ . Stationarizing the action one obtains the solutions of the theory which have finite and localized energy. The solutions are the soliton

$$\psi \equiv \phi_{\text{sol}} = + \frac{\mu}{\sqrt{\lambda}} \tanh \left[ \frac{\mu}{\sqrt{2}} \frac{(x - x_0) - vt}{\sqrt{1 - v^2}} \right], \quad (3)$$

and the antisoliton  $-\psi$ . From the hamiltonian density,  $\mathcal{H} = \frac{1}{2}(\partial_x \phi)^2 + U_s(\phi)$ ,

one can calculate the mass of the soliton and antisoliton

$$m = \int_{-\infty}^{+\infty} dx \mathcal{H}(x) = \frac{2\sqrt{2}}{3} \frac{\mu^3}{\lambda}. \quad (4)$$

One can also define the topological charge,  $Q = \frac{1}{2}[\psi(x = +\infty) - \psi(x = -\infty)]$ , (conserved in time) which has the positive value  $Q_s = +\mu/\sqrt{\lambda}$  in the case of the soliton and the negative value  $Q_{\bar{s}} = -\mu/\sqrt{\lambda}$  in the case of the antisoliton. To this charge one associates the topological current  $k^\mu = \frac{1}{2}\varepsilon^{\mu\nu}\partial_\nu\psi$  which is conserved,  $\partial_\mu k^\mu = 0$ , and such that  $Q = \int_{-\infty}^{+\infty} dx k^0$ .

Now, let us consider a non-degenerate potential  $U$  in action (2) by adding to  $U_s$  a small term that breaks its symmetry [1, 9]:  $U(\phi) = U_s(\phi) + \frac{\epsilon}{2\mu/\sqrt{\lambda}}(\phi - \mu/\sqrt{\lambda})$ , where  $\epsilon$  is the energy density (per unit length) difference between the true ( $\phi_- = -\mu/\sqrt{\lambda}$ ) and false ( $\phi_+ = +\mu/\sqrt{\lambda}$ ) vacua. As noticed in [1, 2, 3],  $\epsilon$  is responsible for both the decay of false vacuum and soliton-antisoliton pair creation.

We want to find an effective one-loop action built from the soliton field itself and that describes the above pair creation process. The soliton field should be a charged field since the system admits two charges,  $Q_s$  and  $Q_{\bar{s}}$ . Therefore, the action should contain the mass term  $m^2\psi\psi^*$ , where  $m$  is the soliton mass given in eq.(4), and the kinetic term  $(\partial_\mu\psi)(\partial^\mu\psi^*)$ . Thus, the free field effective action is  $\int d^2x[(\partial_\mu\psi)(\partial^\mu\psi^*) - m^2\psi\psi^*]$ . However, if one demands local gauge invariance one has to introduce an “electromagnetic” 2-vector potential  $A_\mu$  which transforms the common derivative  $\partial_\mu\psi$  into a covariant derivative  $(\partial_\mu + iQ_s A_\mu)\psi$ . As is well known, the field  $A_\mu$  itself should contribute to the action. This contribution must be gauge invariant since the covariant kinetic term plus the mass term are already gauge invariant. This is achieved by defining the invariant 2-form field,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . In two dimensions, an anti-symmetric field can only be of the form:  $F_{\mu\nu} = \sigma(t, x)\varepsilon_{\mu\nu}$ , where  $\varepsilon_{\mu\nu}$  is the Levi-Cevita tensor and  $\sigma(t, x)$  a function. Therefore the effective one-loop action should be  $S^{\text{eff}} = \int d^2x[(\partial_\nu\psi + iQ_s A_\nu\psi)(\partial^\nu\psi^* - iQ_s A^\nu\psi^*) - m^2\psi\psi^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}]$ .

Note now that the charged soliton acts also as a source, thus modifying the surrounding field. As a first approximation we shall neglect this effect and assume  $A_\mu$  fixed by external conditions. This allows us to drop the contribution of the term  $F_{\mu\nu}F^{\mu\nu}$  in the effective action. Moreover, the external field responsible for the pair creation is essentially represented by the energy

density difference  $\epsilon$  so we postulate that  $F_{\mu\nu}^{\text{ext}} = \frac{\epsilon}{\mu/\sqrt{\lambda}}\varepsilon_{\mu\nu}$ . Therefore,  $A_\mu^{\text{ext}}$  is given by  $A_\mu^{\text{ext}} = \frac{1}{2}\frac{\epsilon}{\mu/\sqrt{\lambda}}\varepsilon_{\mu\nu}x^\nu$ .

Finally, if the system is analytically continued to euclidean spacetime ( $t_{\text{Min}} \rightarrow -it_{\text{Euc}}$ ;  $A_0 \rightarrow iA_2$ ) one obtains the euclidean effective one-loop action for the soliton pair creation problem

$$S_{\text{Euc}}^{\text{eff}} = \int d^2x \left[ |(\partial_\mu - \frac{1}{2}\epsilon \varepsilon_{\mu\nu}x_\nu)\psi|^2 + m^2|\psi|^2 \right]. \quad (5)$$

In the next section this euclidean effective one-loop action is going to be used to calculate the soliton-antisoliton pair production rate (1). Although the calculations are now similar to those found in Affleck, Alvarez and Manton pair creation problem [12], we present some important steps and results since in two dimensions they are slightly different.

### III. Pair production rate

The soliton-antisoliton pair production rate per unit time is equal to the false vacuum decay rate per unit time

$$\Gamma = -2 \text{Im} E_0, \quad (6)$$

where the vacuum energy,  $E_0$ , is given by the euclidean functional integral

$$e^{-E_0 T} = \lim_{T \rightarrow \infty} \int [\mathcal{D}\psi][\mathcal{D}\psi^*] e^{-S_{\text{Euc}}^{\text{eff}}(\psi; \psi^*)}. \quad (7)$$

As it will be verified,  $E_0$  will receive a small imaginary contribution from the negative-mode associated to the quantum fluctuations about the instanton (which stationarizes the action) and this fact is responsible for the decay. Combining (6) and (7) one has

$$\Gamma = \lim_{T \rightarrow \infty} \frac{2}{T} \text{Im} \ln \int [\mathcal{D}\psi][\mathcal{D}\psi^*] e^{-S_{\text{Euc}}^{\text{eff}}(\psi; \psi^*)}, \quad (8)$$

where  $S_{\text{Euc}}^{\text{eff}}$  is given by (5). Integrating out  $\psi$  and  $\psi^*$  in (8) one obtains

$$\Gamma = - \lim_{T \rightarrow \infty} \frac{2}{T} \text{Im tr} \ln [(\partial_\mu - \frac{1}{2}\epsilon \varepsilon_{\mu\nu}x_\nu)^2 + m^2]. \quad (9)$$

The logarithm in (9) can be written as a “Schwinger proper time integral”,  $\ln u = - \int_0^\infty \frac{dT}{T} \exp \left( - \frac{1}{2} u T \right)$ . Taking  $u = [(\partial_\mu - \frac{1}{2} \epsilon \varepsilon_{\mu\nu} x_\nu)^2 + m^2]$ , yields

$$\Gamma = \lim_{T \rightarrow \infty} \frac{2}{T} \text{Im} \int_0^\infty \frac{dT}{T} e^{-\frac{1}{2} m^2 T} \text{tr} \exp \left[ - \frac{1}{2} \left( P_\mu - \frac{1}{2} \epsilon \varepsilon_{\mu\nu} x_\nu \right)^2 T \right]. \quad (10)$$

Notice that now the trace is of the form  $\text{tr} e^{-HT}$ , with  $H = \frac{1}{2} \left[ P_\mu - \frac{1}{2} \epsilon \varepsilon_{\mu\nu} x_\nu \right]^2$  being the Hamiltonian for a particle subjected to the interaction with the external scalar field in a (2+1)D spacetime, and the proper time playing the role of a time coordinate. One has started with a scalar field theory in a euclidean 2D spacetime and now one has found an effective theory for particles in a 3D spacetime. It is in this new context that the pair production rate is going to be calculated. The gain in having the trace in the given form is that it can be written as a path integral  $\text{tr} e^{-HT} = \int [dx] \exp \left[ - \int dT L \right]$ , where  $L = \frac{1}{2} \dot{x}_\mu \dot{x}_\mu + \frac{1}{2} \epsilon \varepsilon_{\mu\nu} x_\nu \dot{x}_\mu$  is the Lagrangian associated with our Hamiltonian. Thus,

$$\Gamma = \lim_{T \rightarrow \infty} \frac{2}{T} \text{Im} \int_0^\infty \frac{dT}{T} e^{-\frac{1}{2} m^2 T} \int [dx] \exp \left[ - \int_0^T dT \left( \frac{1}{2} \dot{x}_\mu \dot{x}_\mu + \frac{1}{2} \epsilon \varepsilon_{\mu\nu} x_\nu \dot{x}_\mu \right) \right]. \quad (11)$$

Rescaling the proper time variable,  $dT \rightarrow \frac{d\tau}{T}$ , and noticing that the path integral is over all the paths,  $x_\mu(\tau)$ , such that  $x_\mu(1) = x_\mu(0)$ , one has

$$\Gamma = \lim_{T \rightarrow \infty} \frac{2}{T} \text{Im} \int [dx] e^{-\frac{1}{2} \epsilon \oint \varepsilon_{\mu\nu} x_\nu dx_\mu} \int_0^\infty \frac{dT}{T} \exp \left[ - \left( \frac{1}{2} m^2 T + \frac{1}{2T} \int_0^1 d\tau \dot{x}_\mu \dot{x}_\mu \right) \right]. \quad (12)$$

The  $T$  integral can be calculated expanding the function about the stationary point  $T_0^2 = \frac{\int_0^1 d\tau \dot{x}^2}{m^2}$ :

$$\int \frac{dT}{T} e^{-f(T)} \sim e^{-f(T_0)} \frac{1}{T_0} \sqrt{\frac{\pi}{\frac{1}{2} f''(T_0)}} \sim e^{-m \sqrt{\int_0^1 d\tau \dot{x}^2}} \frac{1}{m} \sqrt{\frac{2\pi}{T_0}}. \quad (13)$$

Then (12) can be written as

$$\Gamma = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{2}{m} \sqrt{\frac{2\pi}{T_0}} \text{Im} \int [dx] e^{-S_{\text{Euc}}[x_\mu(\tau)]}, \quad (14)$$

where  $S_{\text{Euc}} = m\sqrt{\int_0^1 d\tau \dot{x}_\mu \dot{x}_\mu} + \frac{1}{2}\epsilon \oint \varepsilon_{\mu\nu} x_\nu dx_\mu$ . This integral can be solved using the instanton method. Stationarizing the action, one gets the equation of motion in the (2+1)D spacetime

$$\frac{m\ddot{x}_\mu(\tau')}{\sqrt{\int_0^1 d\tau \dot{x}^2}} = -\epsilon \varepsilon_{\mu\nu} \dot{x}_\nu(\tau') ; \quad \text{with } \mu = 1, 2 \text{ and } \dot{x}_\mu = \frac{dx_\mu}{d\tau} . \quad (15)$$

The instanton,  $x_\mu^{\text{cl}}(\tau)$ , i.e., the solution of the euclidean equation of motion that obeys the boundary conditions  $x_\mu(\tau = 1) = x_\mu(\tau = 0)$  is

$$x_\mu^{\text{cl}}(\tau) = R(\cos 2\pi\tau, \sin 2\pi\tau) ; \quad \text{with } R = \frac{m}{\epsilon} . \quad (16)$$

The instanton represents a particle describing a loop of radius  $R$  in the plane defined by the time  $x_2$  and by the direction  $x_1$ . The loop is a thin wall that separates the true vacuum located inside the loop from the false vacuum outside.

The euclidean action of the instanton is given by  $S_0 = S_{\text{Euc}}[x_\mu^{\text{cl}}(\tau)] = m2\pi R - \epsilon\pi R^2$ . The first term is the rest energy of the particle times the orbital length and the second term represents the interaction of the particle with the external scalar field. The loop radius,  $R = m/\epsilon$ , stationarizes the instanton action. The action is then  $S_0 = \pi m^2/\epsilon$ .

The second order variation operator is given by

$$\begin{aligned} M_{\mu\nu} &\equiv \left. \frac{\delta^2 S}{\delta x_\nu(\tau') \delta x_\mu(\tau)} \right|_{x^{\text{cl}}} = \\ &= \left[ - \left( \frac{m\delta_{\mu\nu}}{\sqrt{\int_0^1 d\tau \dot{x}^2}} \frac{d^2}{d\tau^2} + \epsilon \varepsilon_{\mu\nu} \frac{d}{d\tau} \right) \delta(\tau - \tau') - \frac{m\ddot{x}_\mu(\tau)\ddot{x}_\nu(\tau')}{[\int_0^1 d\tau \dot{x}^2]^{3/2}} \right]_{x^{\text{cl}}} = \\ &= - \left[ \frac{\epsilon}{2\pi} \delta_{\mu\nu} \frac{d^2}{d\tau^2} + \epsilon \varepsilon_{\mu\nu} \frac{d}{d\tau} \right] \delta(\tau - \tau') - \frac{2\pi\epsilon x_\mu^{\text{cl}}(\tau)x_\nu^{\text{cl}}(\tau')}{R^2} . \end{aligned} \quad (17)$$

The eigenvectors  $\eta_\mu^n$ , and the eigenvalues  $\lambda_n$ , associated with the operator  $M_{\mu\nu}$  are such that

$$M_{\mu\nu} \eta_\nu^n(\tau') = \lambda_n \eta_\mu^n(\tau') \delta(\tau - \tau') . \quad (18)$$

From this one concludes that:

(i) the positive eigenmodes are:

- $(\cos 2n\pi\tau, \sin 2n\pi\tau)$  and  $(\sin 2n\pi\tau, -\cos 2n\pi\tau)$  with  $\lambda_n = 2\pi\epsilon(n^2 - n)$ ,  $n = 2, 3, 4, \dots$ ;  
 $(\sin 2n\pi\tau, \cos 2n\pi\tau)$  and  $(\cos 2n\pi\tau, -\sin 2n\pi\tau)$  with  $\lambda_n = 2\pi\epsilon(n^2 + n)$ ,  $n = 1, 2, 3, \dots$ ;  
 (ii) there are two zero-modes associated with the translation of the loop along the  $x_1$  and  $x_2$  directions:  $(1, 0)$  and  $(0, 1)$  with  $\lambda = 0$ ;  
 (iii) there is a zero-mode associated with the translation along the proper time,  $\tau$ :  $(\sin 2\pi\tau, -\cos 2\pi\tau) = -\frac{\dot{x}_\mu^{\text{cl}}}{2\pi R}$  with  $\lambda = 0$ ;  
 (iv) there is a single negative mode associated to the change of the loop radius  $R$ :  $(\cos 2\pi\tau, \sin 2\pi\tau) = \frac{x_\mu^{\text{cl}}}{R}$  with  $\lambda_- = -2\pi\epsilon$ .

Now, we consider small fluctuations about the instanton, i.e., we do  $x_\mu(\tau) = x_\mu^{\text{cl}}(\tau) + \eta_\mu(\tau)$ . The euclidean action is expanded to second order so that the path integral (14) can be approximated by

$$\Gamma \simeq \lim_{T \rightarrow \infty} \frac{1}{T} \frac{2}{m} \sqrt{\frac{2\pi}{T_0}} e^{-S_0} \text{Im} \int [d\eta(\tau)] \exp \left[ -\frac{1}{2} \int d\tau d\tau' \eta_\mu(\tau) M_{\mu\nu} \eta_\nu(\tau') \right]. \quad (19)$$

The path integral in equation (19) is the one-loop factor and is given by  $\mathcal{N}(\text{Det} M)^{-\frac{1}{2}} = \mathcal{N} \prod (\lambda_n)^{-\frac{1}{2}}$ , where  $\lambda_n$  are the eigenvalues of  $M_{\mu\nu}$  and  $\mathcal{N}$  is a normalization factor that will not be needed. To overcome the problem that arises from having an infinite product of eigenvalues, one compares our system with the free particle system

$$\begin{aligned} \int [d\eta] \exp \left[ -\frac{1}{2} \int d\tau d\tau' \eta_\mu(\tau) M_{\mu\nu} \eta_\nu(\tau') \right] = \\ = \int [d\eta] \exp \left[ -\frac{1}{2} \int d\tau d\tau' \eta_\mu(\tau) M_{\mu\nu}^0 \eta_\nu(\tau') \right] \frac{\prod (\lambda_n)^{-\frac{1}{2}}}{\prod (\lambda'_n)^{-\frac{1}{2}}}, \end{aligned} \quad (20)$$

where  $M_{\mu\nu}^0 = -\frac{1}{T_0} \delta_{\mu\nu} \frac{d^2}{d\tau^2} \delta(\tau - \tau')$  is the second variation operator of the free system with eigenvalues  $\lambda'_n = 2\pi\epsilon n^2$ ,  $n = 0, 1, 2, 3, \dots$  (each with multiplicity 4). In equation (20) the first factor is the path integral of a free particle in a (2+1)D euclidean spacetime

$$\int [d\eta] \exp \left[ -\frac{1}{2} \int d\tau d\tau' \eta_\mu M_{\mu\nu}^0 \eta_\nu \right] = \int [d\eta] \exp \left[ -\frac{1}{2T_0} \int d\tau \dot{\eta}_\mu \dot{\eta}_\mu \right] = \frac{1}{2\pi T_0}. \quad (21)$$

In the productory, one omits the zero eigenvalues, but one has to introduce the normalization factor  $\frac{\|dx_\mu^{\text{cl}}/d\tau\|}{\|\eta_\mu^0\|} \sqrt{\frac{1}{2\pi}} = \sqrt{2\pi} R$  which is associated with the



proper time eigenvalue. In addition, associated with the negative eigenvalue one has to introduce a factor of  $1/2$  which accounts for the loops that do expand. The other half contracts (representing the annihilation of recently created pairs) and so does not contribute to the creation rate. So, the one-loop factor becomes

$$\frac{1}{2\pi\mathcal{T}_0} \frac{\prod (\lambda_n)^{-\frac{1}{2}}}{\prod (\lambda'_n)^{-\frac{1}{2}}} = \frac{1}{2\pi\mathcal{T}_0} \frac{1}{2} \frac{i}{\sqrt{2\pi\epsilon}} \sqrt{2\pi} R \frac{\prod_{\lambda>0} (\lambda_n)^{-\frac{1}{2}}}{\prod_{\lambda'>0} (\lambda'_n)^{-\frac{1}{2}}} = i \frac{1}{2\pi\mathcal{T}_0} \frac{1}{2} \sqrt{2\pi\epsilon} \sqrt{2\pi} R. \quad (22)$$

Written like this, the one loop factor accounts only for the contribution of the instanton centered in  $(x_1, x_2) = (0, 0)$ . The translational invariance in the  $x_1$  and  $x_2$  directions requires that one multiplies (22) by the spacetime volume factor  $\int dx_2 \int dx_1 = TL$ , which represents the spacetime region where the instanton might be localized. So, the correct one-loop factor is given by

$$\int [d\eta(\tau)] \exp \left[ -\frac{1}{2} \int d\tau d\tau' \eta_\mu(\tau) M_{\mu\nu} \eta_\nu(\tau') \right] = i \frac{LT}{2\pi\mathcal{T}_0} \frac{1}{2} \sqrt{2\pi\epsilon} \sqrt{2\pi} R. \quad (23)$$

Putting (23) into (19), using  $\mathcal{T}_0^2 = \frac{\int d\tau \dot{x}^2}{m^2} = \frac{(2\pi R)^2}{m^2}$ ,  $R = \frac{m}{\epsilon}$  and  $S_0 = \pi m^2/\epsilon$ , one finally has that the soliton-antisoliton pair production rate per unit time and length is given by

$$\Gamma/L = \frac{\epsilon}{2\pi} e^{-\frac{\pi m^2}{\epsilon}}. \quad (24)$$

We have recovered Stone's exponential factor  $e^{-\frac{\pi m^2}{\epsilon}}$  [1] as well as the prefactor  $A = \epsilon/2\pi$  of Kiselev and Selivanov [2, 3] and Voloshin [4].

Note the difference to the 4D problem of Affleck *et al* [12] and Schwinger [23], who have found for the factor  $A$  the value  $(eE)^2/(2\pi)^3$  which is quadratic in  $eE$  and not linear, as in our case. This difference has to do with the dimensionality of the problems.

It is well known that a one-particle system in 2D can be transformed straightforwardly to a thin line in 3D and a thin wall in 4D, where now the mass  $m$  of the soliton should be interpreted as a line density and surface density, respectively. In fact, a particle in (1+1)D, as well as an infinite line in (2+1)D, can be considered as walls as seen from within the intrinsic space dimension, justifying the use of the name wall for any dimension. Our calculations apply directly to the domain wall pair creation problem in any dimension.

## IV. Conclusions

The equation for the loop of radius  $R$  in 2D euclidean spacetime is given by  $x^2 + t_E^2 = R^2$ , where we have put  $x = x_1$  and  $t_E = x_2$ . One can make an analytical continuation of the euclidean time ( $t_E$ ) to the Minkowskian time ( $t_E = it$ ) and obtain the solution in 2D Minkowski spacetime

$$x^2 - t^2 = R^2. \quad (25)$$

At  $t_E = t = 0$  the system makes a quantum jump and a soliton-antisoliton pair materializes at  $x = \pm R = \pm m/\epsilon$ . After the materialization, the soliton and antisoliton are accelerated, driving away from each other, as (25) shows. To check these statements note first that the energy necessary for the materialization of the pair at rest is  $E = 2m$ , where  $m$  is the soliton mass. This energy comes from the conversion of false vacuum into true vacuum. Since  $\epsilon$  is the energy difference per unit length between the two vacua, we conclude that an energy of value  $E = 2R\epsilon$  is released when this conversion occurs in the region  $(2R)$  within the pair. So, the pair materialization should occur only when  $R$  is such that the energy released is equal to the rest energy:  $2R\epsilon = 2m \Rightarrow R = m/\epsilon$ . This value agrees with the one that has been determined in section III.

After the materialization the pair is accelerated so that its energy is now  $E = 2m/\sqrt{1-v^2}$ . Differentiating (25), we get the velocity  $v = \sqrt{1 - R^2/x^2}$ . The energy of the pair is then given by  $E = 2\frac{m}{R}|x| = \epsilon 2|x|$ . Notice now that  $\epsilon 2|x|$  is the energy released in the conversion of false vacuum into true vacuum. So, after pair creation, all the energy released in the conversion between the two vacua is used to accelerate the soliton-antisoliton pair.

This discussion agrees with the interpretation of the process as being the false vacuum decaying to the true vacuum plus a creation of a soliton-antisoliton pair. It also justifies the presence of the interaction term  $\epsilon \varepsilon_{\mu\nu} x_\nu \psi$  present in the covariant derivative of the proposed effective one-loop action, (5), since  $\epsilon x$  is the energy released in the decay and responsible for the creation and acceleration of the pair.

With the proposed effective one-loop action (5) we have recovered Stone's exponential factor  $S_0$  [1] of the pair creation rate in (1), and the prefactor  $A$  of Kiselev and Selivanov [2, 3] and Voloshin [4]. In the proposed effective one-loop action the soliton charge is treated no longer as a topological charge

but as a Noether charge. Such an interchange between the topological and the Noether charges was already present in [19, 20].

The problem of false vacuum decay coupled to gravity has been introduced in [24] and recently there has been a renewed interest in it (see, e.g., [25, 26]). With the proposed effective one-loop action (5) we pretend to further analyse this problem.

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